

Landau hydrodynamical model at RHIC and LHC

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The rapidity distribution and transverse spectra of most copious particles such as pions, Kaons and antiprotons from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV have been investigated in the framework of Landau hydrodynamical model. With a more realistic choice of freeze-out condition and the employment of lattice equation of state, we find transverse expansion of the collision systems is important to explain the observed data. With the increase of collision energy from RHIC to LHC, transverse flow becomes more and more important for hadron production at midrapidity, especially for more massive particle.

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I. INTRODUCTION

The Relativistic Heavy Ion collider (RHIC) and the Large Hadronic Collider (LHC), the biggest high energy nuclear experiments in the world, realized the creation of a quark gluon plasma (QGP), a thermalized system consist of free quarks and gluons, which was the main matter of our early universe shortly after the Big Bang. In order to understand the organ of our universe and to study the non-perturbative region of quantum chromodynamics, it is of great importance to measure the macroscopic properties of this matter, such as equation of state, transport coefficient, viscosity, phase boundary, and so on. However, direct measurement is extremely difficult because the QGP matter made-in-lab exists only shortly in violent systems, due to the confinement of quarks. In fact, those properties can only be extracted from experimental data based phenomenological models.

Among those models, statistical thermal model was first proposed in 1950's [1], though there was neither the terminology of quark nor that of QGP at that time. This model is still in use today. The basic assumption is, after the creation of a fireball (of QGP), all hadron species are strongly produced under the same temperature followed with weak decays and hadronic interactions. The phase transition temperature T_c is one of the main parameters of the model, which can be extracted from experimental data of particle yields/ratios. The obtained temperature is about 160 MeV, very close to Lattice (LQCD) prediction[2].

However, statistical thermal model can neither describe the dynamics evolution of the collision systems nor cover more observables than particle yields/ratios. Soon Landau [3] proposed hydrodynamical model, which made one step forward. With latter development, more hydrodynamical models have been employed in heavy ion physics, from Bjorken's one-dimensional boost-invariant hydrodynamics to hydrodynamics in two-dimensional

and three-dimensional space, additionally with viscosity and fluctuated initialization. Thus hydrodynamics has become one of the most powerful models, with more and more experimental observables explained, with a more and more realistic but also complicate hydrodynamical evolution. In this paper we will develop the recent work[4, 5], investigate the employment of the oldest and simplest hydrodynamical model, Landau hydrodynamical model, to RHIC and LHC experiments.

The paper is organized as following. In section II, we will introduce our calculation approach of particle production based on Landau model. In section III we will present our results on both rapidity distribution and transverse spectra of the most copious particles such as charged pions, Kaons and antiproton from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, and compare with RHIC and LHC data. The effects from different freeze-out conditions and equation of states will be shown as well. In section IV we will draw the conclusions on what we have learned from this exercise.

II. CALCULATION APPROACH

Hydrodynamics is often employed to describe the evolution of the collision system, from an initial time (initial condition) to a freeze-out time (FO condition). During this period, the system is governed with the energy-momentum conservation law

$$\partial_\mu T^{\mu\nu} = 0. \quad (1)$$

Solving this fluid equation with a given dynamical equation of state (EOS), and decomposing the energy-momentum tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}, \quad (2)$$

one can obtain the energy density ϵ , pressure P , flow velocity u^ν at each space-time point of the collision system. Here we assume an ideal hydrodynamics for the system.

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More term should be included in (2) once viscosity is considered.

The proper time coordinate τ , space-time rapidity coordinate η , r and ϕ are often used to note each space-time point. Thus

$$\begin{aligned} x &= r \cos \phi, \\ y &= r \sin \phi, \\ z &= \tau \sinh \eta, \\ t &= \tau \cosh \eta. \end{aligned} \quad (3)$$

Or inversely

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z}. \quad (4)$$

The initial state of the Landau hydrodynamical model is a thin static disk of thickness Δ and diameter a . It is assumed to be the approximation of the overlap of two highly Lorentz contracted nuclei, and the two geometrical sizes are assumed to be related by

$$\Delta = a/\gamma, \quad (5)$$

with γ the Lorentz factor corresponding to the velocity of the colliding nuclei in the center-of-mass frame,

$$\gamma = \frac{\sqrt{S_{NN}}}{2m_N}, \quad (6)$$

where S_{NN} is the invariant collision energy per colliding nucleons of mass m_N .

The Landau hydrodynamical evolution of the system is divided in two parts: the initial longitudinal expansion (during which transverse velocities and displacements are neglected), and the subsequent "conic flight", where transverse velocities appear and hadronic particles freeze out of the fluid.

Assuming a simple dynamical EOS of the form

$$P = \epsilon/3, \quad (7)$$

an approximate solution of the 1+1 dimensional problem of the longitudinal expansion phase is given by Landau [3]. This solution is summarized (using the notation of [4]) as follow. The energy-density field is

$$\epsilon = \epsilon_0 \exp \left\{ -\frac{4}{3} (y_+ + y_- - \sqrt{y_+ y_-}) \right\}, \quad (8)$$

where y_{\pm} are logarithmic light-cone coordinates

$$y_{\pm} = \ln \frac{t \pm z}{\Delta}, \quad (9)$$

and ϵ_0 is the initial energy-density of the disk. The flow four-velocity are expressed as

$$u^\mu = (\cosh \eta, 0, 0, \sinh \eta). \quad (10)$$

The momentum rapidity of the flow

$$y_{\text{flow}} = \frac{1}{2} \ln \frac{u_0 + u_z}{u_0 - u_z} \quad (11)$$

coincides with the space-time rapidity coordinate η defined in Eq.(4), similarly to the case of the Bjorken model.

Now is the step of "conic flight", with hadronic particles freezing out of the fluid. The four-momentum of each particle can be specified as

$$p^\mu = (m_T \cosh y, p_T \cos \phi_p, p_T \sin \phi_p, m_T \sinh y) \quad (12)$$

in terms of its transverse momentum p_T , rapidity y and azimuth angle ϕ_p , where $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass, with the mass of the particle, m .

The distribution of emitted hadrons in the momentum space is calculated with the Cooper-Frye formula [6]

$$\frac{d^3 N}{dy d^2 p_T} = \int f(x, p) p^\mu d\sigma_\mu, \quad (13)$$

where $f(x, p)$ is the phase-space distribution of particles at freeze-out. The phase-space distribution has the thermal form

$$f(x, p) = \tilde{f} \left(\frac{p^\mu u_\mu(x)}{T(x)} \right) = \frac{g}{(2\pi)^3} \frac{1}{\exp \left(\frac{p^\mu u_\mu(x)}{T(x)} \right) + \xi}, \quad (14)$$

with $\xi=1$ for fermions, -1 for bosons and 0 for classical particles obeying Boltzmann statistics. g is a degeneracy factor that is different for each particles species. The temperature at a given space-time point $T(x)$ is obtained from the energy-density field in eq.(8) with a certain EOS.

In previous work [5], a hadronic gas EOS has been used, where the massive particles in the hadronic phase is assumed to obey the relativistic Boltzmann distribution. Then the pressure of the medium is given by

$$P(T) = \sum_{\alpha} \frac{g_{\alpha}}{\pi^2} m_{\alpha}^2 T^2 K_2(m_{\alpha}/T), \quad (15)$$

where m_{α} and g_{α} are the mass and degeneracy factor of particle type α , and K_2 is a modified Bessel function of the second kind. The energy-density and temperature is related via

$$\epsilon(T) = 3 \sum_{\alpha} P_{\alpha} \left(1 + \frac{m_{\alpha}}{3T} \frac{K_{1\alpha}}{K_{2\alpha}} \right), \quad (16)$$

where we used the notation

$$K_{i\alpha} = K_i(m_{\alpha}/T). \quad (17)$$

In Fig. 1, this relation is plotted with dashed line and note as "Pion", since the lightest mesons contribute the most to the pressure and energy density of hadronic gas. The same relation calculated with quark and gluon degree of freedom has been plotted with solid line and

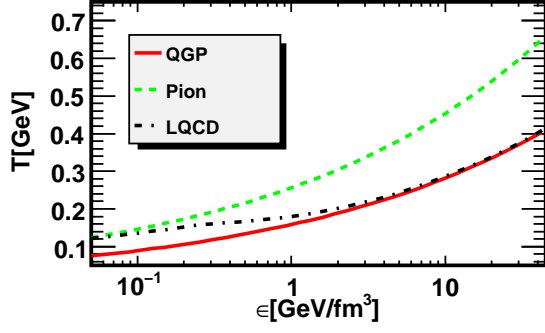


Figure 1: (Color Online) Temperature dependence of energy density. Dashed line: calculated according to Pion gas; Solid line: calculated with QGP freedom; Dotted-dashed line: LQCD result[7].

noted as "QGP". We also plot Lattice result[7] as dotted dashed line and note as "LQCD". One can see Lattice EOS coincides with QGP one at high energy density and coincides with Pion one at low energy density. It is clear that no pions at very high energy density and no QGP at very low temperature, and LQCD result is a more realistic choice. In the following we repeat the previous calculation with Pion EOS and also take LQCD EOS as a comparison.

We will employ Landau fluid dynamics in to RHIC and LHC heavy ion collisions, in the most simple case, central collisions. Therefore, the Landau solution of energy density in eq.(8) is plotted as a function of η and τ in left panel of Fig. 2, where upper panel for the central Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV and lower panel for the central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. The corresponding temperature is plotted in the right panel, based on two choices of EOS.

To calculate hadrons' spectrum with Cooper-Frye formula in eq.(13), we treat the hypersurface element four-vector

$$d\sigma_\mu = (dx dy dz, dy dz dt, dx dz dt, dx dy dt) \quad (18)$$

at each space-time point of FO surface, in terms of τ, η, r and ϕ according to eq.(3) for the convenience of calculation.

Insert $u^\mu u_\mu = 1$ into eq.(13), and make use of eq.(10), one can get

$$p^\mu u_\mu = m_T \cosh(y - \eta) \quad (19)$$

and

$$u^\mu d\sigma_\mu \rightarrow \frac{a^2 \pi}{4} \tau_{FO} d\eta, \quad (20)$$

where the integral over r and ϕ has been done.

In Refs. [3, 4] an estimate of the time of freeze-out is

$$\tau_{FO} = 2a. \quad (21)$$

We take this as a choice of FO condition. Currently the popular FO condition is at a fixed temperature around $T_c \sim 160$ MeV. Here we take a constant energy density ϵ_{FO} as another choice of FO condition, which corresponds to a fixed temperature via EOS. Since FO is a change of the treatment on collision systems, from a collective mode to the collection of many single particles, this change should happen after the appearance of hadrons. Therefore, a realistic FO condition, should not occur at too high energy density where the matter is still in QGP phase. According to Fig. 1, three values 0.08, 0.16 and 0.24 GeV/fm³ were employed for ϵ_{FO} , because Pion EOS and LQCD EOS coincide at $\epsilon \sim 0.08$ GeV/fm³ and 0.24 GeV/fm³ roughly corresponds to $T_C \sim 160$ MeV according to LQCD EOS.

With the FO condition

$$\epsilon = \epsilon_{FO} \quad (22)$$

and the solution of Landau fluid in eq.(8), we can get the freeze-out time at each given space point as the following form,

$$\tau_{FO} = \Delta \exp \frac{4B + \sqrt{4B^2 - 12\eta^2}}{6}, \quad (23)$$

where the constant $B = \frac{3}{4} \ln(\epsilon_0/\epsilon_{FO})$, ϵ_0 the initial energy-density of the disk, Δ the initial thickness of the disk and η the space-time rapidity of the given space point. Then the integral along freeze-out hypersurface can be done easily with Eqs.(19, 20).

III. RESULTS AND DISCUSSION

We first repeated the previous work [5], calculated the rapidity spectra of charged pions from central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV with the same FO condition $\tau_{FO} = 2a$ and the same EOS. The result is shown as a dashed line in Fig. 3, a very nice coincidence with BRAHMS data where empty squares for π^+ and empty circles for π^- . However, this nice coincidence is just an accident. We explain this in the following.

In Fig. 3, the dashed line was decomposed into several dotted lines which are the contributions from different η bin $[-6, -4], [-4, -2], \dots, [4, 6]$, respectively. The energy density and the temperature along η -axis at $\tau_{FO} = 2a$ correspond to the second lines on the lower panel of Fig. 2. As one can see, most midrapidity pions come out from the fluid at $\eta \sim 0$ where the temperature is as high as 300 MeV. This is not realistic because hadrons should not appear at such a high temperature, where the baryonic chemical potential is almost zero. In fact, this energy density is too high to employ Pion EOS, according to Fig. 1.

Then we took LQCD EOS instead. This decreases the temperature at $\tau_{FO} = 2a$ quite a lot, c.f. the right panel of Fig. 2. In Fig. 4, the resulted rapidity distribution of charged pions and charged Kaons, shown as solid lines,

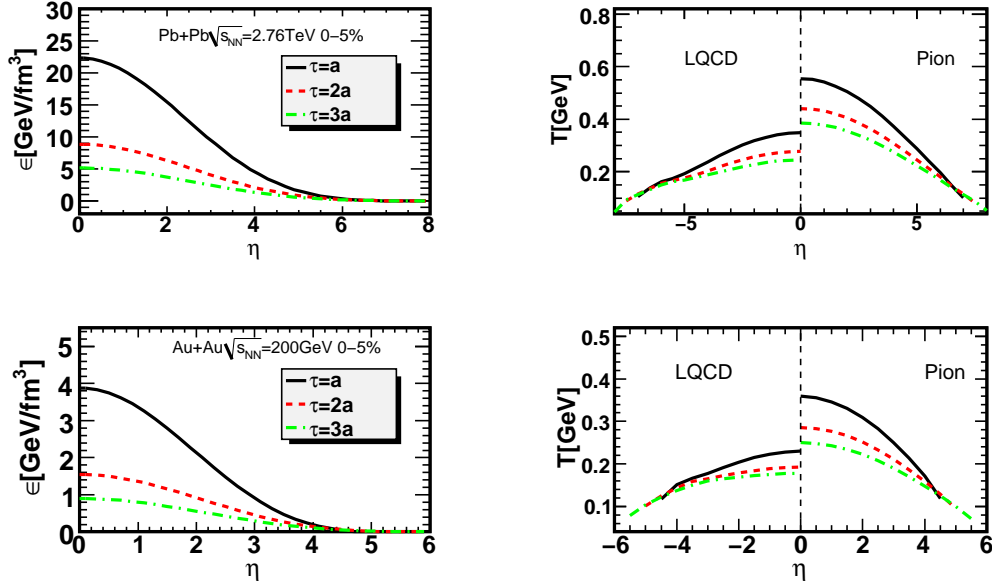


Figure 2: (Color Online) Left panel: energy density dependence of η at different proper time τ where upper panel for central Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV and lower panel for the central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. Right panel: the corresponding temperature as a function of η and τ with two choices of EOS.

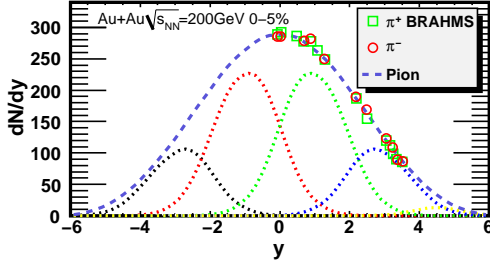


Figure 3: (Color Online) Rapidity distribution of charged pions in central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. Dashed line is calculated with Pion EOS and decomposed dotted lines for the contributions from η bin $[-6, -4]$, $[-4, -2]$, ..., $[4, 6]$, respectively. Data points are from [8].

and Pion EOS results shown as dashed lines, are compared with BRAHMS data points [8]. While Pion EOS results can reproduce BRAHMS measured pion and Kaon rapidity distribution, this realistic LQCD EOS calculation can not!

The reason is clear. At $\tau_{FO} = 2a$, hadrons of large rapidity mainly freeze out from the fluid at large- η , where energy density is low and LQCD EOS coincides with Pion EOS. While at midrapidity, hadrons mainly freeze out from fluid at $\eta \sim 0$ where the energy density is high. Then the temperature interpreted with LQCD EOS is much lower than that with Pion EOS, which makes less hadron emission at midrapidity. As a result, the rapidity shape with LQCD EOS becomes broader and deviates from the measured data. The correct choice of LQCD EOS itself can not reproduce the data, but the deviation

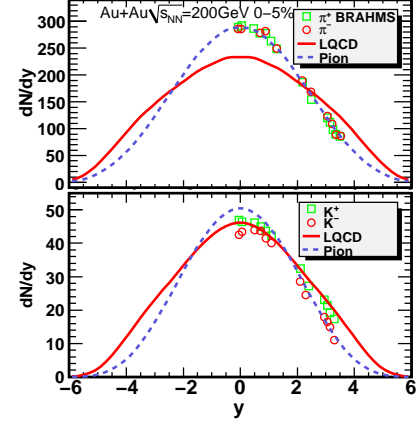


Figure 4: (Color Online) Rapidity distribution of pion (upper panel) and Kaon (lower panel) in central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV, with Pion EOS (dashed line) and LQCD EOS (solid line). Data points are from [8].

can tell us even more.

To understand the deviation better, we calculated the transverse mass spectra of charged pions and Kaons from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, with the FO condition $\tau_{FO} = 2a$ and two options of EOS. The result with Pion EOS is shown in Fig. 5, where solid lines for pions and dashed lines for Kaons, at midrapidity $y = 0$ (left panel) and forward rapidity $y = 3.5$ (right panel) and compared with BRAHMS data where full cycles for π^- and full squares for K^- . The calculation with LQCD EOS is plotted in Fig. 6, where the same notation

has been used. Again, similar results appear at forward rapidity $y = 3.5$ for both EOS, where particles freeze out from quite low energy density source and two EOS coincidence.

However, at midrapidity $y = 0$, the inverse slope of the transverse spectra of both pions and Kaons from LQCD calculation is lower than experimental data, while the calculation with Pion EOS makes higher the inverse slope for pions, due to the extremely high temperature interpreted with Pion EOS.

The second line in lower-right panel of Fig. 2 tells the temperature interpreted with LQCD is about 200 MeV at $\eta \sim 0$, which is lower than Pion EOS value, but still higher than $T_c \sim 160$ MeV. Then Why the calculated inverse slopes of both pions and Kaons are still lower than experimental data? Because there is no transverse expansion in Landau hydrodynamics. Hydrodynamical models with transverse expansion and freeze-out at 160 MeV can make higher inverse slope of transverse spectra and reproduce the data, i.e. [9]. The deviation tell us, the collective transverse expansion of the collision systems is important to explain the measured transverse spectra.

The midrapidity transverse momentum spectra of charged pions, Kaons and antiproton from central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, with the FO condition $\tau_{FO} = 2a$ and two options of EOS are shown in Fig. 7, where solid lines for Pion EOS and dashed lines for LQCD EOS, and ALICE data [10] shown as full cycles. Again we can see Pion EOS gives high inverse slopes than LQCD. The calculated pion slope with LQCD EOS seems to reproduce the data, again by accident. Because the FO condition $\tau_{FO} = 2a$ makes a much higher energy density and the temperature at freeze-out is interpreted as high as 400 MeV, c.f. the second line in the upper panel of Fig. 2. This compensates the missing of transverse expansion in Landau fluid.

In Fig. 7, the deviation of transverse spectra between data and LQCD results, becomes bigger and bigger, as the mass ordering from pion, Kaon to antiproton. This is because, transverse flow drives heavier particles to move along radial direction at the same collective velocity as light particles. However, heavier particles gain more energies and higher inverse slope of the transverse momentum spectra. In another word, transverse expansion of the collision systems becomes more and more important with the increase of hadron mass.

As mentioned above, the FO condition $\tau_{FO} = 2a$ is not realistic, because the energy density and temperature vary a lot along the beam direction. In the following, we take a constant energy density ϵ_{FO} as a FO condition.

In Fig. 8 are shown the transverse mass spectra of pion (upper panel) and Kaon (lower panel) in central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV, where left panel for $y=0$ and right panel for $y=3.5$, with solid line, dashed line and dotted line for $\epsilon_{FO}=0.08, 0.16, 0.24$ GeV/fm³, re-

spectively. The higher ϵ_{FO} can slightly increase the inverse slope of the transverse spectra, while the slopes are still far from the BRAHMS data, due the missing of transverse flow in Landau fluid.

In Fig. 9 are shown the midrapidity transverse momentum spectra of charged pions, Kaons and antiproton from central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, with the FO condition $\epsilon_{FO}=0.08, 0.16, 0.24$ GeV/fm³ shown as solid lines, dashed lines and dotted lines respectively. ALICE data [10] are shown as full cycles. Again, higher ϵ_{FO} makes slightly higher inverse slope. But compared with Fig. 8 for the case of central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV, the miss of transverse expansion of Landau fluid in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is even stronger, for the larger and hotter collision system.

IV. CONCLUSIONS

In summary, we demonstrated the production of most copious particles from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in the framework of Landau hydrodynamical model.

The measured rapidity distribution of pions and kaons from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the transverse momentum spectrum of pions from Central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV can be reproduced with Landau model by accident, due to the choice of FO condition $\tau_{FO} = 2a$ and the employment of Pion EOS. This FO condition makes most hadrons freeze out from a fluid at very high energy density. The employment of Pion EOS causes that the temperature at which hadrons are emitted is interpreted to be as high as 400 MeV. The emission of hadrons at high density and high temperature compensates the missing of transverse expansion in Landau hydrodynamical model.

With a more realistic choice of FO condition, fixed at a relatively low energy density, and the employment of LQCD EOS for particle production, we can see that transverse expansion becomes important to explain bulk hadron production in high energy nuclear collisions. Especially, with the increase of collision energy from RHIC to LHC, and with the increase of the mass of produced hadrons, transverse expansion becomes more and more important.

Acknowledgments

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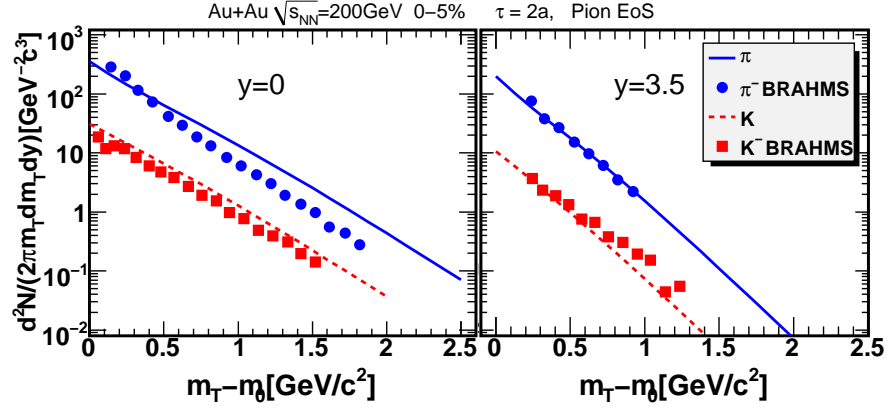


Figure 5: (Color Online) Transverse mass spectra of charged pions and Kaons from central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity $y = 0$ (left panel) and forward rapidity $y = 3.5$ (right panel). The calculation was done with Pion EOS and FO condition $\tau_{FO} = 2a$, where solid lines for pions and dashed lines for Kaons. Circles for π^- and squares for K^- of BRAHMS data [8].

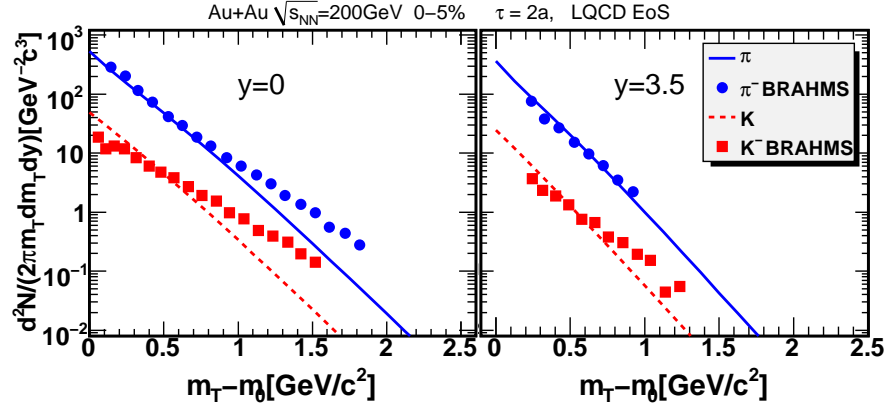


Figure 6: (Color Online) Same as Fig. 5 except the calculation was done with LQCD EOS.

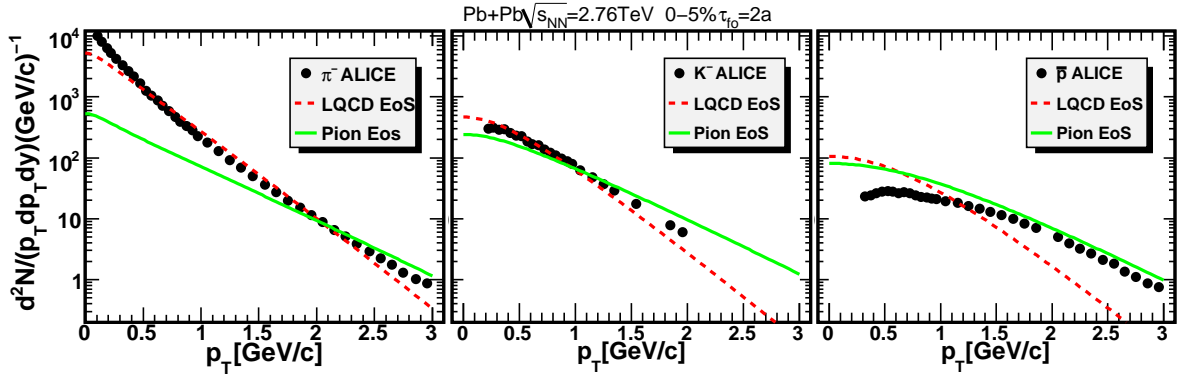


Figure 7: (Color Online) Transverse momentum spectra of pion, Kaon and anti-proton from central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. FO condition $\tau_{FO} = 2a$, solid lines for Pion EOS and dashed lines for LQCD EOS. Data points from ALICE collaboration [10].

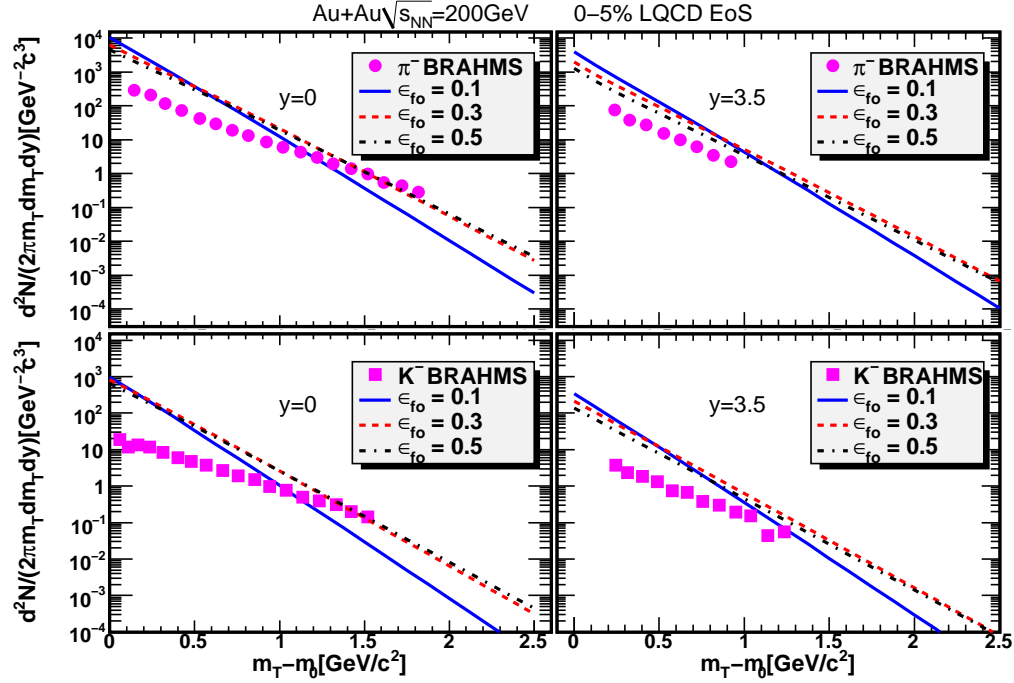


Figure 8: (Color Online) The transverse mass spectra of pion (upper panel) and Kaon (lower panel) in central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV, where left panel for $y=0$ and right panel for $y=3.5$. Solid lines, dashed lines and dotted lines for FO condition $\epsilon_{FO}=0.08, 0.16, 0.24$ GeV/fm³, respectively. Data points are from the BRAHMS collaboration [8].

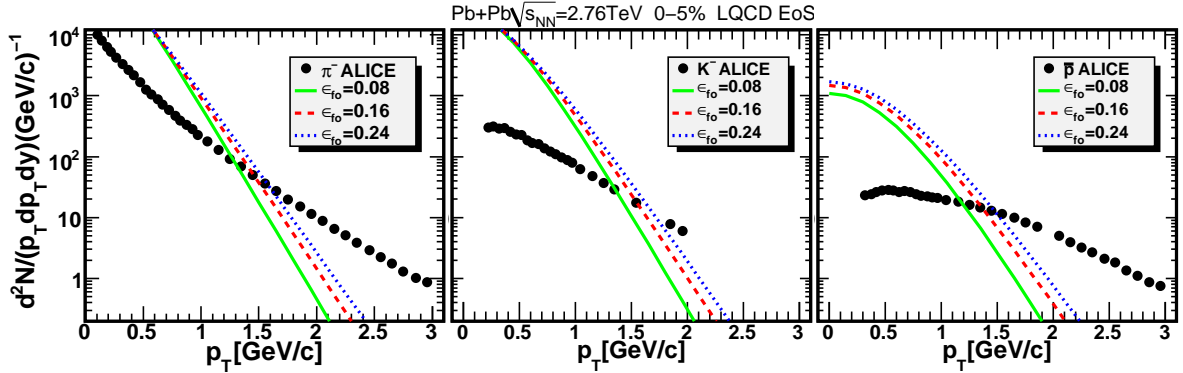


Figure 9: (Color Online) Transverse momentum spectra of pion ,Kaon and anti-proton from central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Solid lines, dashed lines and dotted lines for FO condition $\epsilon_{FO}=0.08, 0.16, 0.24$ GeV/fm³, respectively. Data points from ALICE collaboration [10].

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